Wave Coupling Between Parallel Single-Mode and Multimode Optical Fibers

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***Abstract*—A directional coupler composed of a single-mode optical fiber and a multimode optical fiber has been considered to be capable of serving as a drop/insert device of a node in optical fiber local area networks. It should couple almost all the local transmitter power into the multimode fiber bus, while removing only a small fraction of the optical power from the bus through the single-mode fiber. In this paper, the underlying fundamental process of the power transfer between two such optical fibers is analyzed utilizing a coupled-mode theory. Numerical calculations show that wave coupling among the guided modes on the fibers is quite complicated and that the wave amplitude variations along the propagation direction are different from the sinusoidal types resulting from two-mode coupling. The theoretical results do support the expected performance of the proposed couplers and provide an important guide for the design of such devices.**

I. Introduction

OPTICAL-FIBER TECHNOLOGY is finding increasing utility in various communications, sensor, and signal processing systems. In these systems, various optical components are required to perform different basic functions. Fiber couplers or power dividers are among such components and are used for tapping, dividing, and mixing of optical signals. Several coupler structures, using either multimode or single-mode fibers, based on the well-known phenomenon of evanescent wave coupling, have been reported by various authors [1]–[3]. When the optical-fiber cores are brought into close proximity, wave coupling and thus the transfer of optical power between the fibers can occur due to the interaction of the extended evanescent optical fields just outside of the cores. In the case of two parallel single-mode optical waveguides, it can be shown that the optical power is periodically transferred back and forth between the guides in the approximation of weak coupling [4]. This coupling mechanism can in fact be considered as the beating of two spatially resonant systems, with the spatial beating period or the coupling length determined by the coupling strength between the two interacting waveguide modes. Since the coupling strength depends on the spacing between the fiber waveguides and thus the coupler geometry, theoretical determination of this dependence would greatly help the realization of such devices [3].

Manuscript received March 7, 1986; revised July 8, 1986. This work was supported in part by the National Science Council of the Republic of China under Grant NSC74-0608-E002-06R.

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IEEE Log Number 8610566.

In this paper, we study the evanescent wave coupling between a single-mode and a multimode optical fiber. A directional coupler composed of a single-mode fiber and a multimode fiber finds application in optical-fiber local area networks (LAN's) as the drop/insert (D/I) device of a node [5], [6]. At the node, the receiver picks up a small fraction of the optical energy carried on the fiber bus, and the transmitter injects the lightwave signal onto the bus. It is desirable that the power from the local transmitter be tightly coupled into the bus, while most of the power on the fiber bus can propagate through the node without being taken away. It has been proposed [6] that utilizing a single-mode pigtail fiber for the local transmitter along with the multimode fiber bus may achieve such performance. It is then essential to understand the characteristics of power transfer between two such dissimilar fibers.

We present a theoretical investigation of this problem on the basis of a coupled-mode theory [7]. We assume that the single-mode (SM) fiber and the multimode (MM) fiber are parallel and infinitely long, both with step-index profile. The coupled-mode formulation of the problem is described in Section II. Section III gives the numerical results about the power distribution on both fibers inside the interaction region and discusses the usefulness of these results in the design of a coupler. Section IV deals with the application of our results to real systems.

II. The Coupled-Mode Formalism

We consider two infinitely long and parallel cores embedded in the background cladding, as depicted in Fig. 1. The refractive indices of the cladding, the SM core, and the MM core are denoted as n_c, n_s, and n_m, respectively. The radii of the SM and MM cores and the distance between their centers are shown as a_s, a_m, and D, respectively.

We adopt the coupled-mode equations derived in [7]. For the present system of two parallel lossless fibers, the set of coupled-mode equations can be written as

$$\frac{dA_0(z)}{dz} + i\beta_0 A_0(z) = -i\sum_{k=1}^{N} c_{0k} A_k(z) \tag{1a}$$

$$\frac{dA_k(z)}{dz} + i\beta_k A_k(z) = -ic_{k0} A_0(z), \qquad k = 1,2,\cdots,N. \tag{1b}$$

The z coordinate is taken to be parallel to the fiber axes. In (1a) and (1b), the subscripts 0 and k refer to the SM

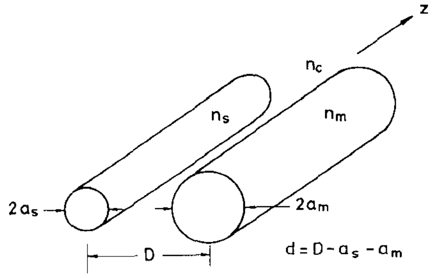

Fig. 1. Sketch of a system of two parallel single-mode and multimode optical fibers.

fiber and the MM fiber, respectively, β_0 and β_k represent the modal propagation constants, and $A_0(z)$ and $A_k(z)$ represent the modal coefficients on the z = constant plane. The symbols c_{0k} and c_{k0} are the coupling coefficients between the guided mode on the SM fiber and the kth guided mode on the MM fiber, and are given by the following surface integrals:

$$c_{0k} = \frac{\omega\epsilon_0}{2}\int_{A_m}\left(n_m^2 - n_c^2\right)\bar{e}_0 \cdot \bar{e}_k \, ds \tag{2a}$$

$$c_{k0} = \frac{\omega\epsilon_0}{2}\int_{A_s}\left(n_s^2 - n_c^2\right)\bar{e}_k \cdot \bar{e}_0 \, ds. \tag{2b}$$

In (2a) and (2b), ω is the wave frequency, ϵ_0 is permittivity of free space, A_s and A_m represent the cross sections of the SM and MM cores, respectively, and $\bar{e}_0$ and $\bar{e}_k$ are the modal electric field vectors. All modes are assumed to travel in the positive z direction.

In writing (1a) and (1b), we have assumed that only the (fundamental) guided mode of the SM fiber and N guided modes of the MM fiber participate in the coupling process. We have also assumed that only forward modes (propagating in the $+z$ direction) are involved in the wave coupling mechanism. It has been shown [7] that the power transfer between forward and backward modes is very small and can be neglected for the case where the coupling coefficients are much smaller than the modal propagation constants, that is, the case satisfying the weak-coupling condition. For the examples discussed in the following section, the c_{0k} and c_{k0} are found to be at least three orders smaller than the accompanying values for β_0 and β_k. We are thus basically considering weak-coupling cases. It was demonstrated [3] that even for relatively closely spaced fiber cores, the weak-coupling approximation holds quite well, based on the good agreement between theoretical calculation and experimental measurements, for the case of two SM fibers.

III. Numerical Results and Discussion

Equations (1a) and (1b) are now employed to analyze the power transfer between step-index SM and MM fibers. The wavelength of the optical signal is taken to be 1.3 μm. The parameters in Fig. 1 are chosen as $n_s = n_m = 1.458$, $n_c = 1.4551$, $a_s = 3.1645$ μm, and $a_m = 25$ μm. A new variable d, defined by $D = a_s + a_m + d$, will be varied to account for different spacings in the following calculations. The linearly polarized modes (LP_{lm} modes) will be used for the guided modes on both fibers. In the present case, the propagation constant β_0 of the guided (LP_{01}) mode on the SM fiber is found to be 7.035485 μm^{-1}. Seventeen guided modes for the MM fiber (i.e., $N = 17$ in (1b)) with β_k close to β_0 are then chosen for the coupled-mode analysis. These modes and their corresponding propagation constants are listed in Table I. Equations (1a) and (1b) thus become a set of 18 equations and are solved numerically.

TABLE I
LP Modes for the Multimode Fiber

k	LP_{lm}	$\beta_k (\mu m^{-1})$
1	LP_{01}	7.046282
2	LP_{02}	7.043941
3	LP_{03}	7.039806
4	LP_{04}	7.034212
5	LP_{11}	7.045435
6	LP_{12}	7.042180
7	LP_{13}	7.037221
8	LP_{21}	7.044325
9	LP_{22}	7.040168
10	LP_{23}	7.034497
11	LP_{11}	7.042970
12	LP_{12}	7.037327
13	LP_{41}	7.041380
14	LP_{42}	7.035485
15	LP_{51}	7.039565
16	LP_{61}	7.037534
17	LP_{71}	7.035295

A. Power Transfer from the SM Fiber to the MM Fiber

The results for the power coupling from the SM core to the MM core are shown in Fig. 2 for $d = 0$, 1, 2, 4, and 8 μm, respectively. The initial conditions are $A_0(0) = 1$ and $A_k(0) = 0$ for $k = 1, 2, \cdots, N$ on the $z = 0$ plane. In all five cases, four of the 17 modes considered for the MM fiber (LP_{04}, LP_{23}, LP_{42}, and LP_{71}) appear to undergo the most significant power coupling and variations, as indicated by the power-versus-distance curves shown in the figure. By comparing β_k's listed in Table I with $\beta_0 = 7.035485$ μm^{-1}, it is found that these four modes in fact have the smallest differences in propagation constant from β_0 among the 17 modes. This confirms the required phase-matching condition for the coupling between or among interacting modes.

It is noticed in Fig. 2(b) that at a distance of 2 mm, almost all the power in the SM fiber has been transferred to the MM fiber. In the cases of Fig. 2(a) and (c), the SM power drops to a minimum at $z = 1.5$ mm and 2.2 mm, respectively. However, the complete power transfer does not occur there. The results of Fig. 2 do show the reasonable relationship between the coupling strength and the rate at which the power transfer takes place. Let us consider the 3-dB distance, or the distance at which the SM power drops to half its initial value. We find that this distance is 0.35 mm for the $d = 0$ μm case and increases monotonically, as the coupling strength decreases, to 1.45 mm for the $d = 4$ μm case. For the $d = 8$ μm case, the coupling is so weak that only 28 percent of the SM power is taken away even at a distance of 5 mm.

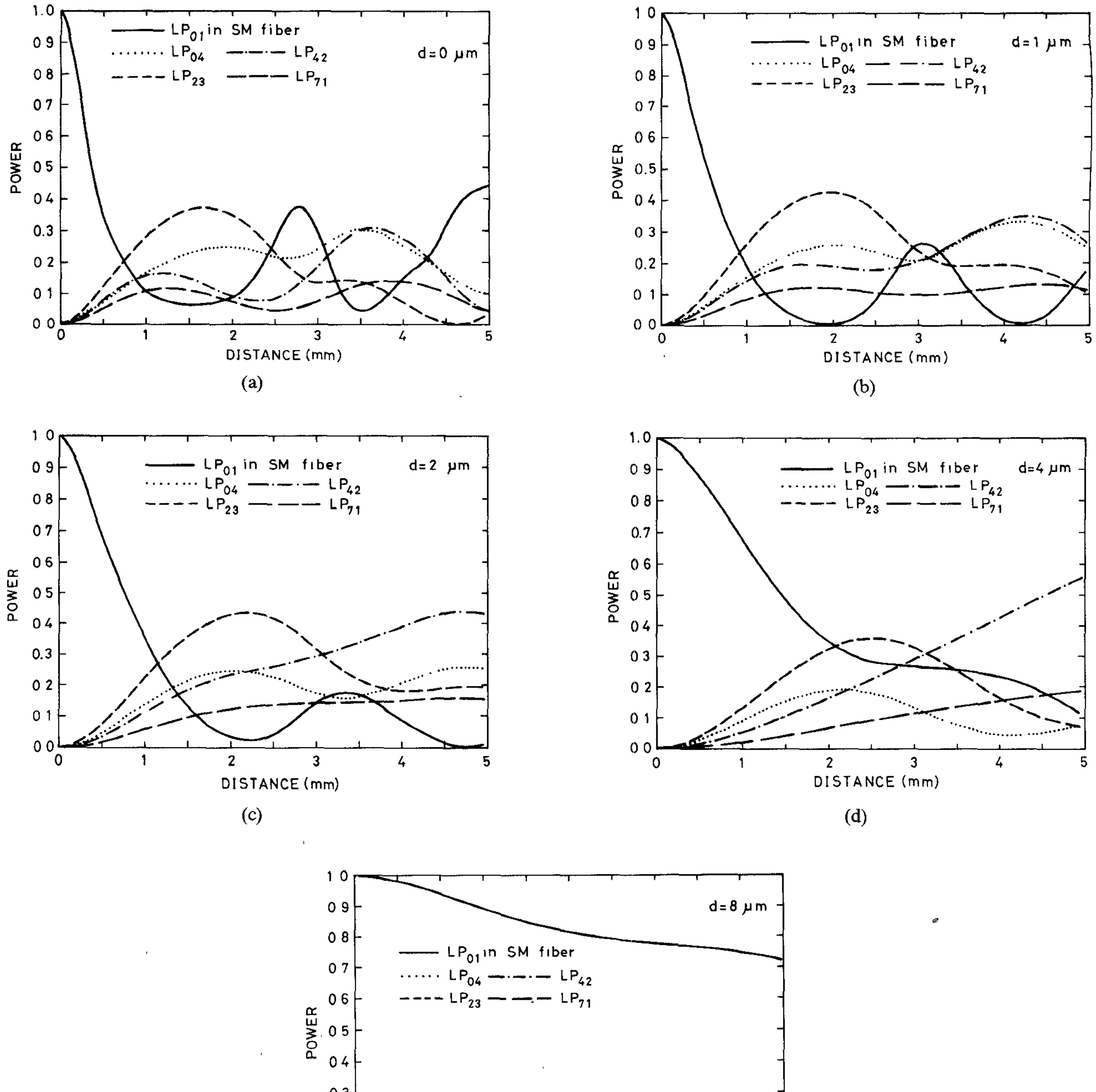

Fig. 2. Power variations versus distance of the fundamental mode on the SM fiber and four guided modes on the MM fiber. The power is initially confined in the SM fiber. Five cases with $d = 0, 1, 2, 4$, and 8 μm, respectively, are shown.

The LP_{23} mode is observed to gain the most power from the SM fiber initially in all five cases. The location where it reaches the maximum value depends obviously on the coupling strength, or the spacing, between the two fibers, as does the 3-dB distance discussed above. Nevertheless, it is found that the LP_{42} mode later on takes this dominant role, as can more easily be seen in Fig. 2(c) and (d). Note that the propagation constant of the LP_{42} mode has almost the identical value as β_0, corresponding to the exact phase-matching condition.

B. Power Transfer from the MM Fiber to the SM Fiber

The reverse situations for the power flow are presented in Fig. 3, again for $d = 0, 1, 2, 4$, and 8 μm. Now the initial conditions are $A_0(0) = 0$ and $A_k(0) = 1$, for $k = 1, 2, \cdots, N$.

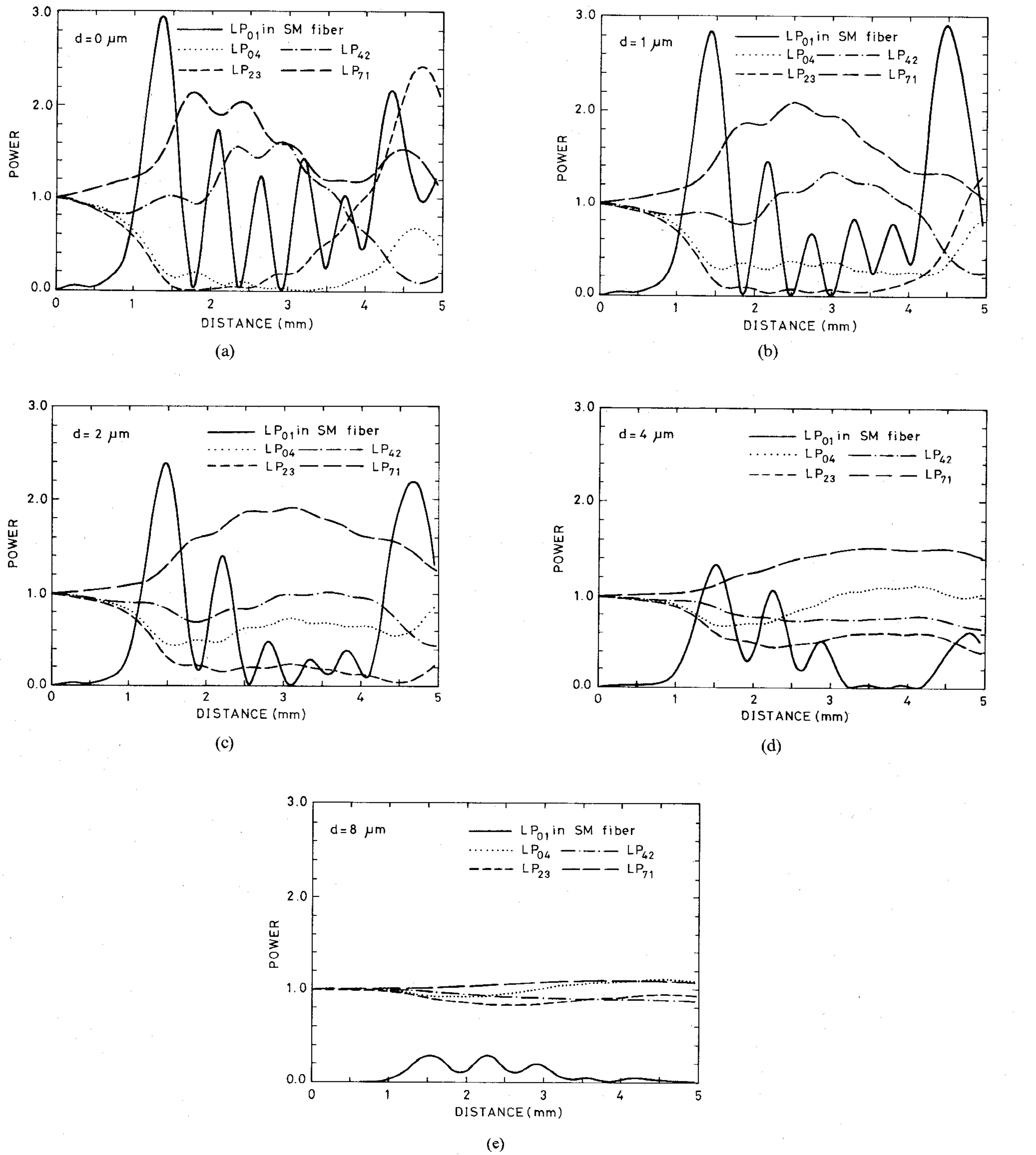

Fig. 3. Power variations versus distance of the fundamental mode on the SM fiber and four guided modes on the MM fiber. The power is initially uniformly distributed over 17 guided modes of the MM fiber. The same five cases as in Fig. 2 are shown.

Note that the total power in each case of Fig. 3 is actually 17 times that in Fig. 2. It is again found that the same four MM modes play dominant roles in the power-coupling and amplitude variations.

Consider the case of $d = 0$ μm. We observe that the first peak power on the SM fiber takes place at $z = 1.35$ mm, with a magnitude equal to 2.95, or 17.4 percent of the total power. It is interesting to note that the location of this peak power on the SM fiber does not change much, or simply moves from $z = 1.35$ mm to 1.5 mm, as d varies from 0 to 8 μm. However, its magnitude does drop gradually to 0.3, or from 17.4 percent to 1.8 percent of the total power in the whole system, as d decreases or as the coupling gets weaker.

Due to the presence of the SM fiber, the mode system of the MM fiber is perturbed. Consequently, power is transferred not only between the SM and MM fibers, but also among the guided modes of the MM fiber itself. Since the MM guided modes are mutually orthogonal, it should be imagined that the power flow among them is via the SM fiber; that is, the power moves from the MM fiber to the SM fiber and returns to the MM fiber. Such mode coupling results in $A_k(z) > 1$ for some modes at certain locations in all five cases. This argument about mode coupling should also apply to those cases given in Fig. 2, although there it is not possible for $A_k(z)$ to be greater than one, i.e., the normalized total system power.

From the above discussion, it is clear that the interaction between the LP_{01} mode on the SM fiber and any guided mode on the MM fiber is strongly affected by the rest of modes on the MM fiber. In other words, the multi-coupled-mode system should be taken into account as a whole whenever mode coupling is concerned. As an example, consider the relation between the LP_{01} and LP_{42} modes, which are exactly phase matched. According to the coupled-mode theory for only two interacting modes with identical propagation constants, the power transfer between these two modes would be sinusoidal and reciprocal [7]. However, the results shown in Figs. 2 and 3 obviously reveal the lack of such features, implying the strong influence of other modes on these two specific coupled modes.

It is interesting to observe the behavior of the LP_{23} and LP_{42} modes for purposes of comparison with our previous discussion in connection with Fig. 2. The LP_{23} mode appears to be an effective power donor at the early stage in the coupling process, being consistent with those cases of power transfer from the SM fiber to the MM fiber. The LP_{42} mode tends to become a more effective donor at a later stage as the power in the LP_{23} mode starts to recover.

C. Coupler Design Considerations

The above results demonstrate that the power coupling between SM and MM fibers is in fact very complicated. It is difficult to describe the power variations in an analytical form, unlike the weak-coupling case involving two interacting modes, which brings about a sinusoidal type of power exchange.

As far as the design of a directional coupler is concerned, we are only interested in a distance of about 1 mm from $z = 0$, which corresponds to the coupling length of a typical integrated optics coupler or fiber coupler [3]. The theoretical curves in Fig. 2 reveal that the power transfer from the SM fiber to the MM fiber increases monotonically and gradually along the propagation direction away from $z = 0$. On the contrary, the power transfer from the MM fiber to the SM fiber as shown in Fig. 3 features a "threshold" type of behavior. More specifically, it takes a distance before the power transfer to the SM fiber starts to increase rapidly toward the first peak value. According to this observation, if we choose a coupling length of 1 mm for the $d = 1$ μm case, 82 percent of the power initially on the SM fiber can be transferred to the MM fiber, while only 2 percent (0.4 versus 17) of that on the MM fiber is taken away and moved to the SM fiber. This meets the asymmetrical power exchange requirements for application as a D/I device in the nodes of local networks mentioned in the introduction. Figs. 2 and 3 thus provide an important guide for such design. For example, the theoretical results tell that, for a fixed coupling length, say 1 mm, the percentage of power flow from the SM to the MM fiber can be adjusted by changing the spacing d. At the same time, the characteristic that the power taken away from the MM fiber is kept relatively low is maintained.

IV. Some Thoughts on Real Systems

In addition to the above coupler design considerations, we would like to make the following remarks concerning the application of the coupling mechanism described here to real systems.

A. Mode Mixing

The SM–MM directional coupler studied in this paper is designated as a mode-selective coupler in [6], where it was conjectured for simplicity that the SM fiber was coupled appreciably to only one or a few of the modes of the MM fiber. It was suggested that some mode-scrambling device be put between nodes so that the power reaching the next node could be made sufficiently uniform among the modes of the MM bus and so that the mode-selective characteristic could effectively be maintained.

Our numerical model can provide a test for the necessity of mode-scrambling between nodes. The problem is equivalent to examining how the coupling effects are influenced by the initial conditions at $z = 0$. Consider the case shown in Fig. 3(a) as an example, where $A_k(0) = 1$ for $k = 1, 2, \cdots, N$. Assume that the coupling length is 1 mm and the interaction between the SM and MM fibers ceases at $z = 1$ mm (by some practical means, say, the fibers are curved and the effective interaction length is finite [3]). The magnitudes of A_k ($z = 1$ mm) are not identical now and they can be used as the initial conditions at $z = 0$ of the coupler at the next node if we assume that these N modes retain their power Until reaching the next node, marked node 2. Similarly, we can calculate the power removed from the MM fiber to the SM fiber at node 3 using the magnitudes of A_k ($z = 1$ mm) of node 2 as the initial conditions, and so on at the following nodes. We have found that the power variation versus distance of the SM fiber at each node retains characteristics similar to those shown in Fig. 3(a); e.g., the locations of power maxima and minima do not change, except that the power level decreases from node to node. Fig. 4 shows the percentage of the initial power on the MM fiber at node 1 (i.e., 17) obtained by the SM fiber at $z = 1$ mm at each node. The percentage is 4.3 percent at node 1, that is, 0.73 versus 17, as can be calculated from Fig. 3(a), and drops to 2.7 percent at node 2. Therefore, the nonuniformity in power distribution among those MM modes resulting from the interaction at node 1 has caused a significant reduction

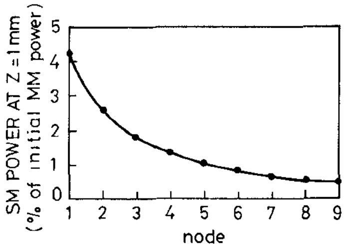

Fig. 4. Percentage of the initial power on the MM fiber at node 1 obtained by the SM fiber at $z = 1$ mm at each node. The initial conditions at each node are defined to be as those at $z = 1$ mm at the previous node.

in coupling efficiency at node 2. If the mode power distribution were made to be sufficiently uniform, the SM power at $z = 1$ mm at node 2 would be only slightly less than 4.3 percent since the values of the initial conditions were only slightly smaller than those of node 1. This discussion strongly suggests that mode-scrambling devices be used.

B. Graded-Index MM Fibers

Practical systems often contain MM optical fibers of graded-index cores. Quantitative determination of the effect of different graded-index distributions on the dynamics of coupling with a SM fiber is more involved than the step-index case. However, from the viewpoints of coupling strength and phase matching, the results of the step-index case should be representative of coupling behavior in standard systems, as discussed in the following.

The mode field is confined better for the graded-index MM fiber than for the step-index guide, assuming that the propagation constants are the same. To achieve a given coupling strength, the SM core might have to be put closer to the graded-index core compared with the case with a step-index MM core, since small d results in greater coupling strength according to Figs. 2 and 3. Therefore, we might have to choose a negative d, i.e., $D < a_s + a_m$ to obtain a satisfactory coupling whenever the graded-index fiber is involved.

As for the phase matching, we expect that whatever MM fiber is chosen, the SM fiber should be suitably designed so that its propagation constant is within the range of β_k for the MM modes. The propagation constants for the MM guided modes used in the previous section range from 7.034497 μm^{-1} to 7.046282 μm^{-1}, as listed in Table I, and that of the SM mode is 7.035485 μm^{-1}. To see the role of phase matching, we consider four cases, with β_k for the SM mode being 7.038651 μm^{-1}, 7.040247 μm^{-1}, 7.046283 μm^{-1}, and 7.050514 μm^{-1}, respectively, and with the MM fiber unchanged. The core radii are 4.5 μm and 5.4 μm, respectively, for the first and second cases. For the third and fourth cases, the core radii are chosen to be 3.85 μm and 3.5 μm, respectively, and the core indices are raised a little such that the normalized frequency is kept below 2.405 and the fiber works as a SM guide. Assuming $A_k(0) = 1$ and $A_k(0) = 0$ $(k = 1, 2, \cdots, N)$, the power variations on the SM fiber for these four cases are shown in Fig. 5. Here, we take $d = 0$ μm and these results should be compared with the case of Fig. 2(a).

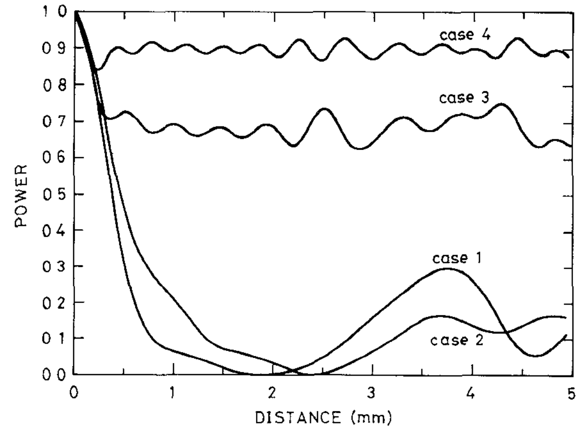

Fig. 5. Power variations versus distance of the fundamental mode on the SM fiber for four cases with different propagation constant β_0. The β_0's for cases 1, 2, 3, and 4 are 7.038651 μm^{-1}, 7.040247 μm^{-1}, 7.046283 μm^{-1}, and 7.050514 μm^{-1}, respectively.

Fig. 5 clearly demonstrates the importance of phase matching on the energy coupling. For cases 1 and 2, β_0 is matched to the range of β_k, and coupling is strong, as for the case of Fig. 2(a). In case 3, the phases are barely matched and the coupling has become much weaker. When the phases are further out of match, as in case 4, the coupling becomes so inefficient that only a very small portion of the SM power can be transferred to the MM fiber.

C. Curved Structures

In a real fiber coupler, the fibers are often curved [3]. Consequently, the spacing between the fibers, and therefore the coupling coefficient, are a function of the position along the interaction region. The theory of distributed coupling as developed in [3], [8], and [9] should be used in order to make a more accurate prediction of the power transfer, especially along the typical coupling length up to about 1 mm. This consideration is currently under investigation and will be reported in the future.

V. Summary

This paper has described the power coupling between a single-mode optical fiber and a step-index multimode optical fiber, utilizing a coupled-mode analysis. The numerical results show that the power transfer among different modes in such two-waveguide systems can be fairly complicated. Nevertheless, they confirm that a directional coupler composed of a SM fiber and a MM fiber can meet the requirements of a drop/insert device needed in a local area network, as proposed elsewhere [6]. Thus, the theoretical curves provided would be useful for the design of such couplers.

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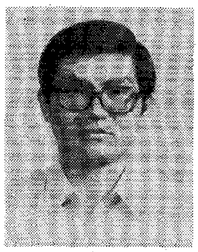

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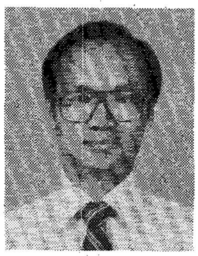

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